MATH 54 - MOCK MIDTERM 2

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Name: ____

Instructions: This is a mock midterm, designed to give you extra practice for the actual midterm. Good luck!!!

1		50
2		20
3		20
4		10
5		10
6		10
7		15
8		10
9		20
10		15
Total		180

Date: Monday, October 24th, 2011.

1. *(50 points, 5 pts each)*

Label the following statements as **T** or **F**.

Make sure to **JUSTIFY YOUR ANSWERS!!!** You may use any facts from the book or from lecture.

(a) If A and B are square matrices, then $det(A+B) = det(A) + det(B)$.

(b) If $\mathcal{A} = \{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$ and $\mathcal{D} = \{\mathbf{d_1}, \mathbf{d_2}, \mathbf{d_3}\}$ are bases for V, and P is the matrix whose ith column is $[\mathbf{d_i}]_{\mathcal{A}}$, then for all $\mathbf{x}$ in V, we have $[\mathbf{x}]_{\mathcal{D}} = P\,[\mathbf{x}]_{\mathcal{A}}$

(c) If $Nul(A) = \{\mathbf{0}\}$, then A is invertible.

(d) A 3×3 matrix A with only one eigenvalue cannot be diagonalizable

(e) $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$

(f) If W is a subspace of V and $\mathcal{B}$ is a basis for V, then some subset of $\mathcal{B}$ is a basis for W.

(g) If $\mathbf{v_1}$ and $\mathbf{v_2}$ are 2 eigenvectors corresponding to 2 **different** eigenvalues λ_1 and λ_2, then $\mathbf{v_1}$ and $\mathbf{v_2}$ are linearly independent!

(h) If a matrix A has orthogonal columns, then it is an orthogonal matrix.

(i) For every subspace W and every vector $\mathbf{y}$, $\mathbf{y} - Proj_W \mathbf{y}$ is orthogonal to $Proj_W \mathbf{y}$ (proof by picture is ok here)

(j) If $\mathbf{y}$ is already in W, then $Proj_W \mathbf{y} = \mathbf{y}$

2. *(20 points)* Find a diagonal matrix D and an invertible matrix P such that $A = PDP^{-1}$, where:

$$A = \begin{bmatrix} 7 & -6 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 7 \end{bmatrix}$$

3. *(20 points)* Use the Gram-Schmidt process to obtain an orthonormal basis of $W = Span\left\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\right\}$, where:

$$\mathbf{v_1} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} 6 \\ -3 \\ 1 \\ 11 \end{bmatrix}$$

4. *(10 points)* Find the determinant of the following matrix A:

$$A = \begin{bmatrix} 1 & 42 & 536 & 789 & 4201 & 123456789 \\ 0 & 1 & 2011 & 2012 & \pi m & \text{Dolphin} \\ 0 & 0 & 2 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 2 & -1 \end{bmatrix}$$

Note: The answer may surprise you :)

5. *(10 points)* Find a least squares solution to the following system $A\mathbf{x} = \mathbf{b}$, where:

$$A = \begin{bmatrix} 2 & 0 \\ 1 & -1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

6. *(10 points)* Define $T : P_3 \to P_3$ by:

$$T(p(t)) = tp''(t) - 2p'(t)$$

Find the matrix A of T relative to the basis $\mathcal{B} = \{1, t, t^2, t^3\}$ of P_3

7. *(15 points)* Let $\mathcal{B} = \left\{ \begin{bmatrix} 7 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}, \mathcal{C} = \left\{ \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \end{bmatrix} \right\}$.

(a) Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$

(b) Find $[\mathbf{x}]_{\mathcal{C}}$, where $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

8. *(10 points)* Find the orthogonal projection of t^2 onto the subspace W spanned by $\{1, t\}$, with respect to the following inner product:

$$\langle p, q \rangle = \int_{-1}^{1} p(t)q(t)dt$$

9. *(20 points, 10 points each)*

(a) Find a basis for $Row(A)$ and $Col(A)$, where:

$$A = \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 0 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix}$$

(b) What is $Rank(A)$? What is $Dim(Nul(A))$?

10. *(15 points)*

(a) Find an invertible matrix P and a matrix C of the form $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ such that $A = PCP^{-1}$, where:

$$A = \begin{bmatrix} 2 & -2 \\ 1 & 0 \end{bmatrix}$$

(b) Write C as a composition of a rotation and a scaling.